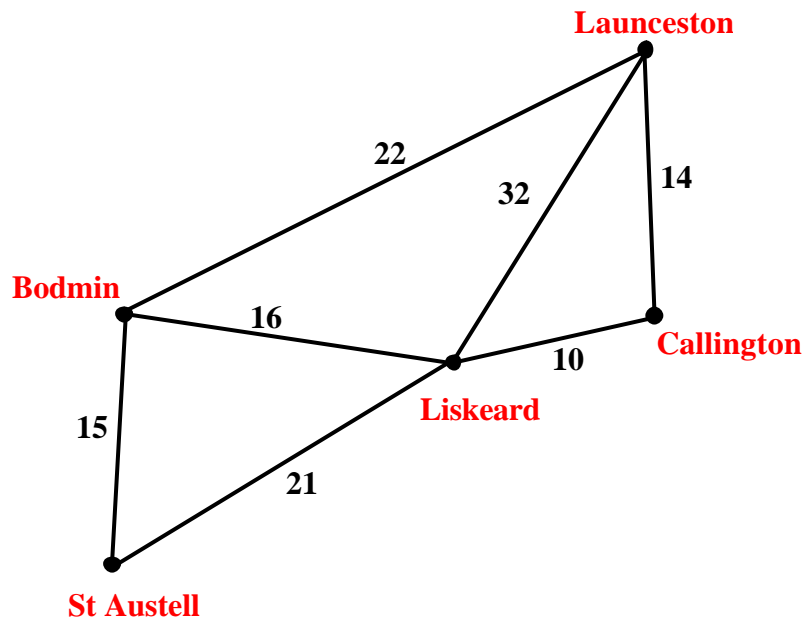


Shortest Path

The network and matrix below give estimated times in minutes to travel along roads joining some places in Cornwall.



Time matrix

	B	St	Li	C	La
B	0	15	16	0	22
St	15	0	21	0	0
Li	16	21	0	10	32
C	0	0	10	0	14
La	22	0	32	14	0

Suppose you want to travel from St. Austell to Launceston as quickly as possible. In a simple network such as this, it is easy to see that the route via Bodmin will be the quickest. But in more complex networks it is often much more difficult to select a quickest or shortest route.

One method that will always work is called Dijkstra's Algorithm. This is summarised below:

Dijkstra's Algorithm

- Step 1** Give the permanent label 0 to the starting vertex.
- Step 2** Give temporary labels to each vertex that is connected directly to the starting vertex.
- Step 3** Find the vertex with the smallest temporary label, number it and make it permanent.
- Step 4** Give temporary labels to each vertex that is connected directly to the previous vertex. Then find the vertex with the smallest temporary label, number it and make it permanent.
- Step 5** Repeat Step 4 until you have given a permanent label to the finishing vertex.
- Step 6** Use the permanent labels to trace back through the network to find the shortest path.

How Dijkstra's algorithm gives the solution to the example above is shown overleaf.



Diagram 1

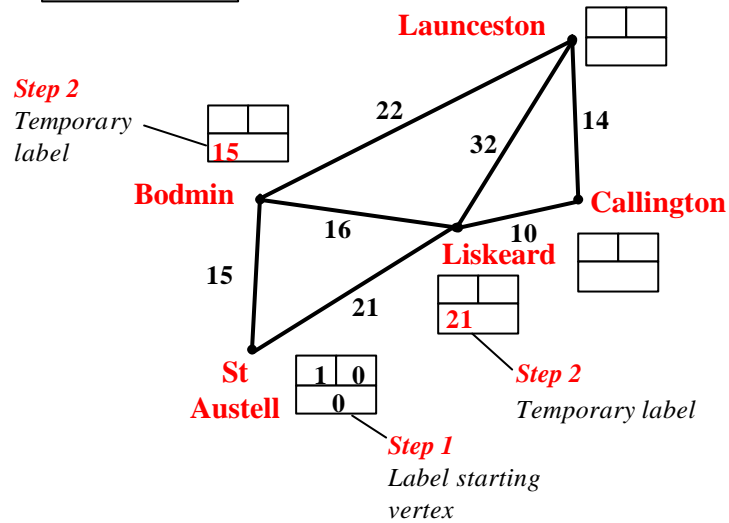


Diagram 2

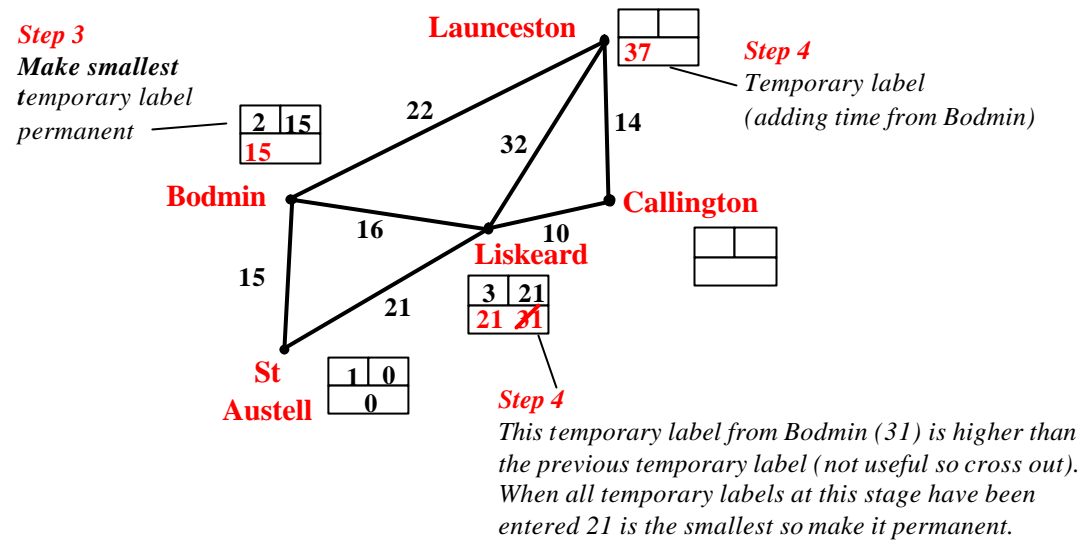


Diagram 3

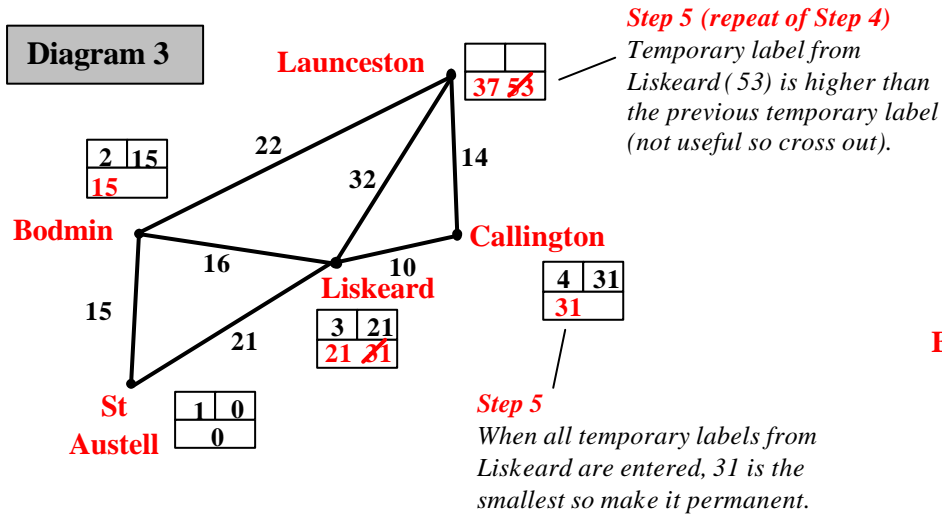
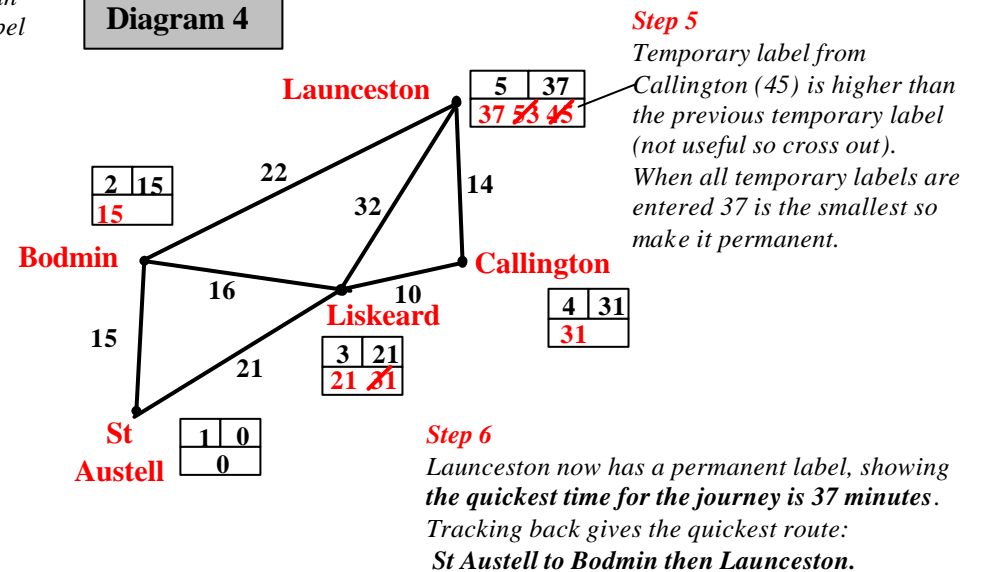


Diagram 4



Shortest Path

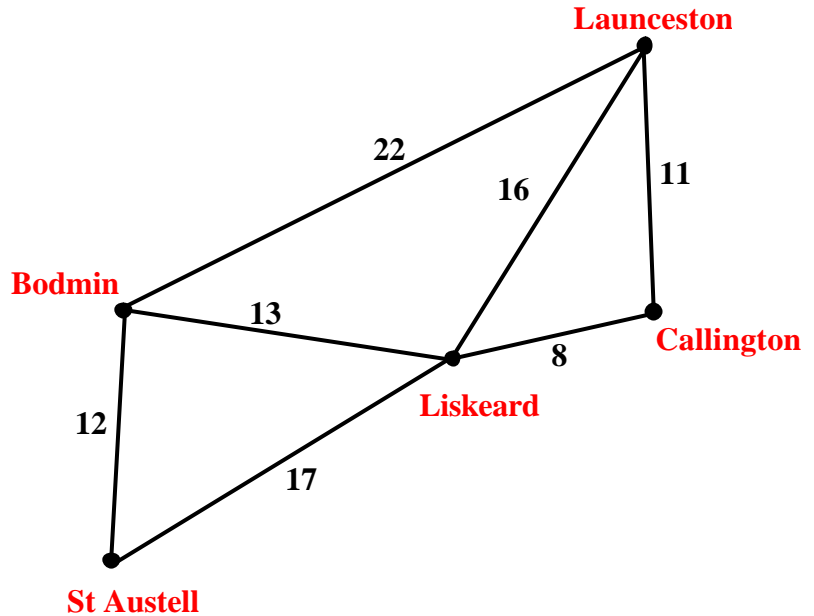
Worksheet

Shortest Route

The road network and **distance** matrix (in miles) for the places in Cornwall are shown below.

Distance matrix

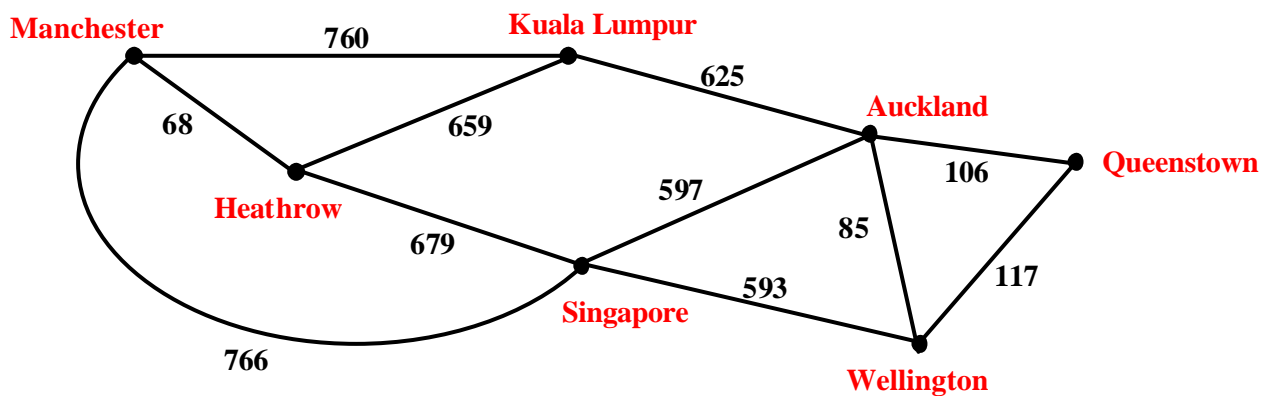
	B	St	Li	C	La
B	0	12	13	0	22
St	12	0	17	0	0
Li	13	17	0	8	16
C	0	0	8	0	11
La	22	0	16	11	0



Suppose you wish to use the **shortest** route from St. Austell to Launceston (rather than the quickest route). Use Dijkstra's algorithm to find the shortest route.

Cheapest Airfare

The network below gives some of the air routes that can be travelled between the UK and New Zealand. The weights on the arcs show return fares in UK pounds.



- Use Dijkstra's algorithm to find the cheapest route from Manchester in the UK to Queenstown in New Zealand.
- What other considerations might affect the decision on which route to take?



Teacher Notes

Unit Advanced Level, Using and applying decision mathematics

Notes on Activity

Pages 1 and 2 introduce Dijkstra's algorithm for finding the shortest path in a network and shows how this can be used to find the quickest route from St. Austell to Launceston. The Powerpoint presentation can be used to give a step by step explanation.

The last Powerpoint slide introduces the first example on the worksheet (Page 3) which uses distances rather than time. The second example on the worksheet involves airfares.

Answers**Shortest Route**

The shortest route from St. Austell to Launceston is via Liskeard, a total of 33 miles.

Cheapest Airfare

- a) The cheapest route is: Manchester – Heathrow – Singapore – Auckland – Queenstown.
The total cost by this route is £1450.
- b) There are many other considerations that may affect the decision and these will depend on the purpose of the journey. Possible suggestions include flight times, the number of stops, the overall time taken, the possibility of breaking the journey to spend some time at the places visited (for holiday or business purposes) etc.

